Pseudo-Harmonic Representation of Gravity Field over South China Sea

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Abstract. Recently, the marine mean gravity anomaly dataset are available to us on a 2 arc minute grid obtained from satellite altimeter data. To formulate gravity field model based on this dataset, in this work we refer to use the method named Pseudo-Harmonic Regional Analysis (PHRA) instead of the conventional Spherical Harmonic Analysis (SHA) due to its high resolution for describing the regional gravity field. The result show that even if taking factor f_{sc}=5 in PHRA, a model to degree 1080 will be enough to represent mean gravity anomaly dataset with the accuracy ±2.4 mGal on 2 arc minute grid, the RMS of the differences between the PHRA model and the geoid undulations altimeter derived are ±0.095m. The PHRA model offers significant improvements compared with any existing models with the same degree and order.

Keywords. Tailored model, Pseudo-Harmonic Regional Analysis, Altimetry, Gravity anomaly

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1 Introduction

The spherical harmonic expansion is a powerful method to describe local and global gravity field in the frequency domain. In principle, the resolution and precision of gravity field expressed by the spherical harmonic model are proportional to the degree and order of model expressed, for example, to degree 5400 that has a resolution about 3.6 km on the equator. For this reason a higher degree geopotential model is of the advantage for us. From the geopotential model we can easily calculate any gravity field quantity, such as geoid undulations, gravity anomalies, vertical deflections, disturbance gravity anomalies and gravity gradients etc. on variant spatial resolutions.

Only the long wavelength parts of the gravity field can be determined from satellite to satellite tracking. Combining the long wavelength parts with terrestrial gravity and altimeter data, which reveal the short wavelength character, produced a complete geopotential model. During the past ten years, a number of the high resolution and accurate global geopotential models have been developed to degree and order 360, for example OSU91A1F (Rapp and Pavlis 1990). To the best of author's knowledge N.K.Pavlis had developed a higher resolution global geopotential model to degree and order 500. In particular, the currently most accurate high resolution global geopotential model EGM96 (Lemoine et al. 1998) has been computed by a collaboration of NASA, US National Imagery and Mapping Agency (NIMA) and Ohio State University (OSU). It seems more difficult to compute a global higher degree geopotential model such as to above 1800 degrees. The main limitations are lack of homogeneous data in space, time and precision within global area. However, it is possible to develop a higher degree regional geopotential model in terms of tailored method on regional area (Wenzel 1998).

Presently global marine gravity anomalies with the high accuracy and high spatial resolution, such as 2 arc minutes, have been derived from satellite altimeter (SA). It is possible to develop ultra high degree regional geopotential models on the sea area. To compute a regional geopotential model with a 2 arc-minutes spatial resolution, it should be developed to 5400 degree and order by means of spherical harmonics expansions (tailored method). Generally, it would be difficult to compute such geopotential models by double accuracy computer and high-powered computing facilities should be necessary in order to perform the intensive computations with the huge amount and variety of data involved and to improve the numerical stability of super-high degree harmonics. To cope with this situation, Belikov et al. (1993) was introduced the Pseudo Harmonic Regional Analysis (PHRA) method. The PHRA leads to increase of the resolution of the regional geopotential model. On the basis of the PHRA principle, using a scale factor as f_{sc}=5 and expanding to 1080 degree and order it corresponds to the spherical harmonics expansions to 5400 degree and order i.e. with a 2 arc-minute spatial resolution. As an example we compute the PHRA model named SCS-P to 1080 degree and

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order by means of the PHRA method with a factor as $f_{sc}=5$ using Sandwell's version 9.2 grid of marine gravity anomalies at 2 arc-minute resolution recovered from altimeter data (Sandwell 1997) over the South China Sea (105-122ºE, 0-25ºN).

2 Method of PHRA Model Computation

In order to compute the PHRA model, a tailored model will be calculated using the tailored method refer to a global geopotential model firstly (Weber 1988. Hsu et al. 1995). Then the PHRA model could be computed basis on the tailored model.

2.1. Tailored Model

The disturbing potential T is represented by a finite number of trial harmonic functions L_{nm} (Rapp and Pavlis 1990).

$$T = \frac{GM}{R}\sum_{n=2}^{N}\sum_{m=-n}^{n}\overline{C}_{nm}L_{nm}(r,\phi,\lambda) \tag{1}$$

The residual disturbing potential are represented as follows

$$\Delta T = \frac{GM}{R}\sum_{n=2}^{N}\sum_{m=-n}^{n}\Delta\overline{C}_{nm}L_{nm}(r,\phi,\lambda) \tag{2}$$

Where

$$L_{n,\binom{2m}{2m+1}}(r,\phi,\lambda) = \left(\frac{R}{r}\right)^{n+1}\overline{P}_{nm}(\sin\phi)\begin{cases}\cos m\lambda\\ \sin m\lambda\end{cases} \tag{3}$$

and $\Delta\overline{C}_{nm}$ are corrections with respect to harmonic coefficients $\overline{C}_{nm}$ of the reference global geopotential model (e.g. EGM96, Lemoine 1998) that are caused by regional residual gravity field $\delta\Delta\overline{g}$. The $\Delta\overline{C}_{nm}$ can be represented as follows (Hsu et al. 1995),

$$\Delta\overline{C}_{nm}(\delta\Delta\overline{g}) = \frac{1}{4a\pi\gamma}\sum_{t=1}^{T}r_t^E\sum_{k=0}^{s}\frac{L_{nmk}(r,\phi,\lambda)}{\overline{S}_{n-2k,|m|}(\frac{b}{E})} \cdot\frac{\overline{IP}_{n-2k,|m|}}{(n-2k-1)q_{n-2k}^t}\delta\Delta\overline{g}_t\begin{cases}IC_t & if \quad m\geq 0\\ IS_t & if \quad m<0\end{cases} \tag{4}$$

$$\overline{IP}_{n|m|} = \int_{\delta_i}^{\delta_{i+1}}\overline{P}_{n|m|}(\cos\delta)\sin\delta d\delta$$

$$IC_j = \int_{\lambda_j}^{\lambda_{j+1}}\cos\lambda d\lambda\ ,\quad IS_j = \int_{\lambda_j}^{\lambda_{j+1}}\sin\lambda d\lambda$$

Where $\overline{S}_{nm}$ is related to the associated Legendre function of the second kind (Gleason 1988), q_n is smoothing factor, s is the greatest integer less than or equal to $\frac{1}{2}(n-|m|)$, and T is the total amount blocks in local area.
We have the tailored model as follows,

$$\overline{C}_{nm}^{tailo} = \overline{C}_{nm}^{EGM96} + \Delta\overline{C}_{nm}w_n \quad \text{for n,m=2-360}$$
$$\overline{C}_{nm}^{tailo} = \Delta\overline{C}_{nm}w_n \quad \text{for n,m=361-1080} \tag{5}$$

The weight function W_n was selected as follows (Basic 1989),

$$W_n = \begin{cases} 0 & n < N_{min}\\ (n-N_{min})/(N_{max}-N_{min}) & N_{min}\leq n\leq N_{max}\\ 1 & N_{max} < n\end{cases} \tag{6}$$

where $N_{\min}$ is the lowest degree that should be kept in initial model and $N_{\max}$ is the starting degree that should be fully corrected.

2.2 Pseudo-Harmonic Regional Analysis (PHRA) Model

In the Pseudo Harmonic Regional Analysis (or locally harmonic) trial functions associated with solid spherical harmonics (3) were introduced as follows (Belikov et al. 1993),

$$L_{n,\binom{2m}{2m+1}}(h,\phi,\lambda) = \left(\frac{R}{R+f_{sc}h}\right)^{n+1}\overline{P}_{nm}(\sin\phi_1)\begin{cases}\cos m\lambda_1\\ \sin m\lambda_1\end{cases} \tag{7}$$

Where f_{sc} is the 'scaling' factor, characterizing the spatial enlargement of the regional domain under consideration in accordance with isomorphism.

$$\phi_1 = \phi_0 + f_{sc}(\phi-\phi_0),\ \lambda_1 = \lambda_0 + f_{sc}(\lambda-\lambda_0)$$
$$h_1 = f_{sc}h$$

The equation (2) and (4) would be rewrite as

$$\Delta T^{PHRA} = \frac{GM}{R}\sum_{n=2}^{N}\sum_{m=-n}^{n}\Delta\overline{C}_{nm}^{PHRA} L_{nm}(h,\phi_1,\lambda_1) \qquad (8)$$

$$\Delta\overline{C}_{nm}^{PHRA}(\delta\Delta g) = \frac{1}{4a\pi\gamma}\sum_{t=1}^{T} r_t^{E} \sum_{k=0}^{s} \frac{L_{nmk}(h,\phi_1,\lambda_1)}{\overline{S}_{n-2k,|m|}(\frac{b}{E})} \frac{\overline{IP}_{n-2k,|m|}}{(n-2k-1)q_{n-2k}^{t}}\delta\Delta\overline{g}_t^{Tailo}\begin{cases}IC_t & if \quad m \ge 0\\ IS_t & if \quad m < 0\end{cases} \qquad (9)$$

The scaling procedure leads to the increase of the resolution of ΔT, roughly equal to its half-wavelength, from the value $180/N_{max}$ to $180/(f_{sc}N_{max})$, with the factor f_{sc} not greater than π/Δ_{max}, Δ_{max} means the maximum linear size of the regional domain (Belikov, 1993). The coefficients of the PHRA with weight power function will be presented by

$$\overline{C}_{nm}^{PHRA} = \Delta\overline{C}_{nm}^{PHRA} w_n \quad \text{for n,m=2-1080} \qquad (10)$$

Where $\Delta\overline{C}_{nm}^{PHRA}$ are the computed coefficients by the PHRA, and $\delta\Delta g^{Tailo}$ mean regional residual gravity anomalies as well as short-wavelength parts. Both caused by the tailored model. We have PHRA model SCS-P to degree and order 1080 as follows,

$$\overline{C}_{nm}^{SCS-P} = \overline{C}_{nm}^{tailo} + \overline{C}_{nm}^{PHRA} \qquad (11)$$

Choosing f_{sc} as 5, then spatial resolution will be $180/(f_{sc}N_{max})$ = 2', while N_{max} were 1080.

3. Computed Results and Comparisons

3.1 The Data Used

We choose a research area over the South China Sea and its vicinity (105-122ºE, 0-25ºN) to compute the model namely SCS-P to 1080 degree and order by means of the PHRA method. The initial marine gravity anomaly data come from Sandwell's version 9.2 (Sandwell and Smith 1997). The Sandwell's version 9.2 of marine gravity anomalies is the most recent one in his series using GEOSAT, ERS-1 and Topex/Poseidon data. The Sandwell's version 9.2 is available at 1 or 2 arc-minute resolutions from the http://topex.ucsd.edu/marine_grav/mar_grav.html. Only a 2 arc-minute grid is used in this work.

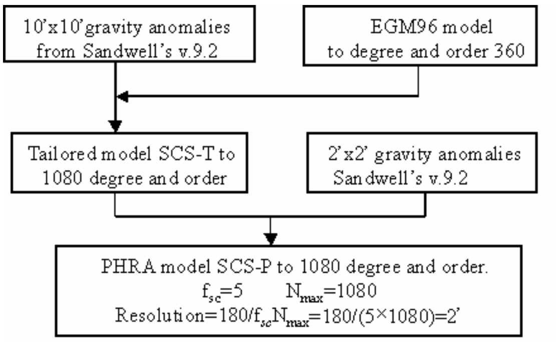

Fig.1 The Flow Chart Computing the PHRA Model SCS-P to Degree and Order 1080

3.2 Computing the PHRA Model SCS-P

The flow chart computing the PHRA model SCS-P shows as in Fig.1. Firstly, we made a 10'×10' grid of marine gravity anomaly from the Sandwell version 9.2 at a 2 arc-minute resolution to compute the tailored model SCS-T to 1080 degree and order based on the global geopotential model EGM96. Then, the PHRA model SCS-P was computed based on the tailored model SCS-T using 2' ×2' grid marine gravity anomaly (ibid) to 1080 degree and order with a factor of f_{sc}=5. As mentioned above, the PHRA model SCS-P corresponds to spherical harmonic expansions to 5400 degree and order i.e. with a 2 arc-minute spatial resolution.

3.3 Comparing the Results

The geopotential models obtained have been compared with initial altimeter-derived marine gravity anomalies and geoid undulations, given by D.T.Sandwell in the research area. The Sandwell's geoid was only as a reference for model comparison. Table 1 shows the differences between the initial gravity anomalies recovered from SA (ibid) and those computed from models (in mGal). The gravity anomalies computed from the PHRA model SCS-P to degree and order 1080 achieve a root mean square error (RMS) of ±2.4 mGal with a spatial resolution about 2', while the tailored model SCS-T to degree and order 1080 achieve a RMS of ±5.4 mGal with a spatial resolution about 10'. The reference model EGM96 has a RMS of ±15.5 mGal with a spatial resolution about 30'. The global model GPM98C (Wenzel 1998) has a RMS of ±9.9 mGal truncated to 1080 degrees with a spatial resolution about 10'. The differences between PHRA model SCS-P and initial gravity anomalies are distinctly small comparing with the tailored model SCS-T and reference model EGM96. That means, the PHRA model SCS-P offers significant improvements on accuracy when compared with other models.

Table 1. Differences (in mGals) between Initial Gravity Data and those Computed from Models.

Model	RMS	Min	Max.	Degree	Resolution
EGM96	15.5	-132.0	135.6	360	30’
GPM98C	9.9	-82.9	104.8	1080	10’
SCS_T	5.4	-39.5	43.5	1080	10’
SCS_P	2.4	-17.5	19.2	1080	2’

Table 2. Comparisons of the Geoid Undulation Computed by D.T.Sandwell and Computed from Models (in Meter)

Model	RMS	Min	Max.	Degree	Resolution
EGM96	0.241	-1.703	1.804	360	30’
GPM98C	0.181	-1.416	2.103	1080	10’
SCS_T	0.093	-0.846	1.258	1080	10’
SCS_P	0.095	-0.665	1.037	1080	2’

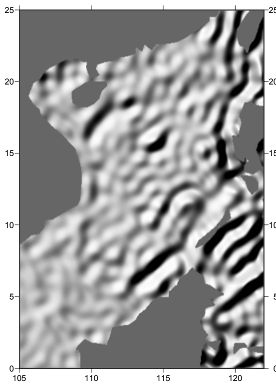

Fig.2a Gravity Anomalies Computed from EGM96 Model to Degree 360

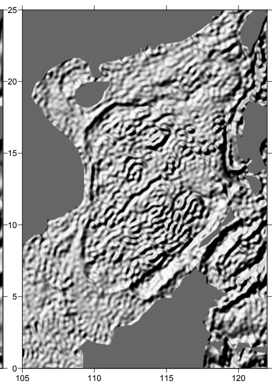

Fig.2b Gravity Anomalies Computed from Tailored Model SCS-T to Degree 1080

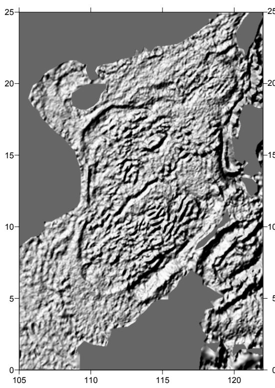

Fig.2c Gravity Anomalies Computed from PHRA Model SCS-P to Degree 1080

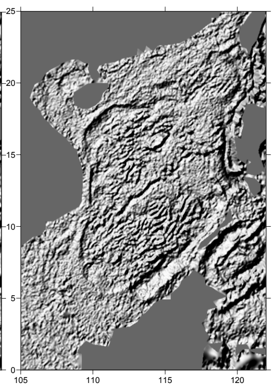

Fig.2d Initial Gravity Anomalies on a 2’x2’ Grid Derived from SA (Sandwell 1997)

Fig.2a-d show shaded relief map of gravity anomaly from models and SA data, respectively. The Fig.2a shows roughly gravity field on the research area with a spatial resolution about 30’. The Fig.2b is better than the Fig.2a, and inferior to the Fig.2c. The Fig.2c, computed from PHRA model SCS-P with a spatial resolution about 2’, shows a fine gravity field that distinctly similar to the Fig.2d, altimeter-derived initial marine gravity anomalies on 2’×2’ grid (Sandwell, 1997). The PHRA model SCS-P offers significant improvements on resolution when compared with tailored models and reference global model.

Table 2 shows the comparisons of geoid undulation computed by D.T.Sandwell and those computed from models (in meter). The geoid undulation computed from the PHRA model SCS-P achieve a RMS of the differences of ± 0.095 m with a spatial resolution about 2’, the tailored model SCS-T achieve a RMS of ± 0.093 m with a spatial resolution about 10’, while the original model EGM96 has a RMS of ± 0.241 m with a spatial resolution about 30’. The global model GPM98C (Wenzel 1998) has a RMS of ± 0.181m truncated to 1080 degrees with a spatial resolution about 10’. The RMS of PHRA model SCS-P is the same as that of tailored model, however, the differences between PHRA model SCS-P and reference geoid are small than the tailored model SCS-T and model EGM96. The PHRA model SCS-P provides a higher spatial resolution than the tailored model SCS-T and reference model EGM96.

4 Conclusions

Global marine gravity anomalies with the high accuracy and spatial resolution have been derived from satellite altimetry. To develop ultra high degree regional geopotential model on the sea area is possible. However it would be difficult by spherical harmonics expansions, and is necessary to employ new modeling methods. As a new modeling method, the PHRA method (Pseudo -Harmonic Regional Analysis) can improve the accuracy and spatial resolution of the regional geopotential model. Moreover computer time is reduced distinctly. Based on the PHRA method, we computed the PHRA model SCS-P to degree and order 1080 with a spatial resolution about 2 arc-minutes in the research area over the South China Sea and its vicinity (105-122^{o} E, 0-25^{o} N). The result show that taking the factor f_{sc}=5 in PHRA, the model to degree 1080 will be enough to represent mean gravity anomaly dataset with the accuracy ± 2.4

mGal on 2 arc minutes grid, otherwise it needs the expansion to degree 5400 by SHA expansion. Experimentally, the PHRA model SCS-P can fit the regional gravity field well and have a higher spatial resolution than other models.

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